1) $\cos 2\theta > \cos\theta - \frac{1}{2}$. This simplifies to $2\cos^2\theta - \cos\theta - \frac{1}{2} > 0$. Solving this yields $\cos\theta < \frac{1-\sqrt{5}}{4}$ or $\cos\theta > \frac{1+\sqrt{5}}{4}$, or $\theta\epsilon[0,\frac{\pi}{5}) \cup (\frac{3\pi}{5},\frac{7\pi}{5}) \cup (\frac{9\pi}{5},2\pi]$. $\frac{6\pi/5}{2\pi} = \frac{3}{5}$. $A = 3$ and $B = 5$, so $A + B = 8$.

2) The maximum height Brilli achieves on day n is $3n + 2$. The smallest n for which the inequality $3n + 2 \geq 50$ is satisfied is $n = 16$.

3) The original solution has 7.2 L HCl. If x L of 5% HCl is added, then the portion of new solution that is HCl is $\frac{7.2+0.05x}{24+x}$. This is equal to $\frac{1}{10}$ when $x = 96$.

4) The minute hand is 210° from the 12. The hour hand is $\left(90 + 35 \cdot \frac{30}{60}\right) = 107.5^\circ$ from the 12. This is a difference of 102.5°.

5) Trevor's displacement is 5 miles south and 12 miles west, for a magnitude of 13 miles. He travels a distance of 21 miles. This is a difference of 8 miles.

6) Let $P(T)$ be the probability that the test labels someone as a nerd, and let $P(N)$ be the probability that someone is a nerd. Trivially, $P(N) = 0.1$ and $P(T|N) = 0.98$. $P(T) = P(T|N) + P\left(T\middle|N'\right) = 0.98 \cdot 0.1 + 0.03 \cdot 0.9 = 0.125$. According to Bayes's Theorem, $P(N|T) = \frac{P(N)\cdot P(T|N)}{P(T)} = \frac{0.1\cdot 0.98}{0.125} = 0.784$.

7) $-i = \text{cis}\frac{3\pi}{2}$. Thus, the sixth roots of $-i$ are $\text{cis}(\frac{\pi}{4} + \frac{k\pi}{3})$ for $k \in \{0, \ldots, 5\}$. The sum of these angles is $\frac{3\pi}{2} + 5\pi = \frac{13\pi}{2}$.

8) In the time between Bjergsen passing Svenskeren and Svenskeren beginning to run, Bjergsen runs 18 meters. Svenskeren runs 3 meters per second faster than Bjergsen and thus catches up to Bjergsen in 6 seconds. This means he runs 72 meters.

9) $n = n^2 + 5n + 4$ simplifies to $n^2 + 4n + 4 = 0$. Thus, $n = -2$.

10) Waffles swims 20 mph upstream and 60 mph downstream. Thus, Waffles swims 40 mph in still water and is in a 20 mph current.

11) The first term implies $x < 9$. The second term is only defined if $x^2 - 6x - 16$ and $3^x - 2^x$ are both less than zero or both greater than zero (or the fraction is equal to 0). The former is positive when $x < -2$ or $x > 8$. The latter is positive when $x > 0$ and negative when $x < 0$. Thus, the domain of the function is $[-2,0) \cup [8,9)$. This domain has length 3.

12) The extreme (here, the minimum) of a quadratic occurs at $-\frac{b}{2a}$. $b = -100$ and $a = 25$, so the minimum occurs 2 afters after 9:00 PM, or at 11:00 PM.

13) Investigation of the formula for multiplying matrices yields that the sum of the entries of the product of two square matrices is equal to the 1-by-1 matrix formed by constructing a row vector with the sum of the entries of each column of the first matrix and multiplying it by the column vector with the sum of the entries of each row of the second matrix. Here, this computation is $\begin{bmatrix} 1 & -1 & 0 & 5 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 4 \\ -7 \\ 2 \end{bmatrix} = [-30]$.

14) In one hour, Tom paints $\frac{1}{8}$ of the fence. The amount that Huck Finn paints in an hour can be calculated by solving $\frac{1}{\frac{1}{8}+h} = 2$, or $h = \frac{3}{8}$. The amount that Joe Harper paints in an hour can be calculated by solving $\frac{1}{\frac{1}{8}+j} = 6$, or $j = 24$. Finally, the amount of time Huck Finn and Joe Harper would take to paint the fence is $\frac{1}{\frac{3}{8}+\frac{1}{24}} = \frac{12}{5}$ hours $= 144$ minutes.

15) Rotate the conic by using the standard formulas $x = x' \cos\theta - y' \sin\theta$, $y = x' \sin\theta + y' \cos\theta$, and $\cot 2\theta = \frac{A-C}{B}$. It is found that $\cot 2\theta = \frac{3}{4}$, so $\cos\theta = \frac{2}{\sqrt{5}}$ and $\sin\theta = \frac{1}{\sqrt{5}}$. Simplifying the expression $4\left(\frac{2x'}{\sqrt{5}} - \frac{y'}{\sqrt{5}}\right)^2 - 4\left(\frac{2x'}{\sqrt{5}} - \frac{y'}{\sqrt{5}}\right)\left(\frac{x'}{\sqrt{5}} + \frac{2y'}{\sqrt{5}}\right) + 7\left(\frac{x'}{\sqrt{5}} + \frac{2y'}{\sqrt{5}}\right)^2 = 24$ yields $3x^2 + 8y^2 = 24$, or $\frac{x^2}{8} + \frac{y^2}{3} = 1$. The area of the ellipse is $ab\pi = \pi\sqrt{24} = 2\pi\sqrt{6}$.

16) Emily wins if she flips two heads or if she flips two heads after some number of consecutive fails by herself and RJ. Since each set of flips is independent, the sequence of probabilities that Emily wins on the nth round is geometric. Let p be the probability that a given flip lands heads. The probability that Emily wins is $p^2 + (1-p^2)(1-p)p^2 + (1-p^2)^2(1-p)^2p^2$ $+(1-p^2)^3(1-p)^3p^2 + \cdots = \frac{p^2}{1-(1-p^2)(1-p)} = \frac{p}{-p^2+p+1}$. Setting this equal to $\frac{1}{2}$ yields $p = \phi - 1$. The probability that RJ wins if he goes first is $p + (1-p)(1-p^2)p + (1-p)^2(1-p^2)^2p + (1-p)^3(1-p^2)^3p + \cdots = \frac{p}{1-(1-p)(1-p^2)} = \frac{1}{-p^2+p+1}$. This is equal to $\frac{1}{2p}$. Substituting back in $p = \phi - 1$, the probability RJ wins if he goes first is $\frac{1}{\sqrt{5}-1} = \frac{1+\sqrt{5}}{4} = \frac{\phi}{2}$.

17) A solution not involving Fermat's Two-Square Identity with Cauchy-Schwartz is shown. Let $a^2 + b^2 = c^2 + d^2 = k^2$ for some constant k. Invoking trigonometry, let $a = k\sin\alpha$, $b = k\cos\alpha$, $a = k\sin\beta$, and $a = k\cos\beta$. Substituting into the expression, the fraction $\frac{20k^2\cos\alpha\sin\beta + 18k^2\cos\alpha\cos\beta + 20k^2\sin\alpha\cos\beta - 18k^2\sin\alpha\sin\beta}{2k^2}$ is obtained. This simplifies to $10\cos\alpha\sin\beta + 9\cos\alpha\cos\beta + 10\sin\alpha\cos\beta - 9\sin\alpha\sin\beta$. Noting the similarity to angle addition formulas, this is equal to $10(\cos\alpha\sin\beta + \sin\alpha\cos\beta) + 9(\cos\alpha\cos\beta - \sin\alpha\sin\beta) = 10\sin(\alpha+\beta) + 9\cos(\alpha+\beta)$. Setting $\gamma = \alpha + \beta$ and noting that the maximum value of $m\sin\theta + n\cos\theta$ is $\sqrt{m^2+n^2}$, the square of the maximum value of $10\sin\gamma + 9\cos\gamma$ is 181.

18) Solving the system of equations $b + c = 49$ and $1.5b + 0.5c = 52.5$ yields $b = 28$ and $c = 21$.

19) Steven's dorm and the library are the foci of an ellipse.

20) Captain Kirk's movement can be modeled as the sum of three geometric sequences, one parallel to the x-axis, one at a 60° angle, and one at a 120° angle. The distance parallel to the x-axis is $\frac{1}{2} - \frac{1}{16} + \frac{1}{128} - \cdots = \frac{\frac{1}{2}}{1+\frac{1}{8}} = \frac{4}{9}$. The distances at the 60° and 120° angles can be found by dividing by 2, obtaining $\frac{2}{9}$ and $\frac{1}{9}$ respectively. The sum of the polar coordinates $\left(\frac{4}{9}, 0^\circ\right)$, $\left(\frac{2}{9}, 60^\circ\right)$, and $\left(\frac{1}{9}, 120^\circ\right)$ is the Cartesian point $\left(\frac{1}{2}, \frac{\sqrt{3}}{6}\right)$. $A = 1$, $B = 2$, $C = 3$, and $D = 6$, so $A + B + C + D = 12$.

21) Using the formula that the number of zeroes at the end of $n!$ is equal to $\sum_{i=1}^{\infty}\left\lfloor\frac{n}{5^i}\right\rfloor$, it can be approximated that the minimum value of k such that $n!$ ends in k zeroes is $4k$. Substituting $n = 8072$ into the formula yields that $8072!$ ends in 2014 zeroes. The final four zeroes come from multiplying by 8075, 8080, and 8085, so $n = 8085$, and the sum of the digits of n is 21.

22) WLOG, let $a \geq b > 0$. Counting cases, $b = 1$ yields $a \in \{1,2,3,4,5\}$, $b = 2$ yields $a \in \{2,3,4\}$, and $b = 3$ yields $a \in \{3,4\}$. Multiplying these cases by 8 yields 80 cases in the nonzero integers. Making sure not to double-count $(0,0)$, there are 9 cases where at least one of a and b equals 0, for a total of 89 cases.

23) This problem is analogous to inserting balls into urns, where urns represent the position of a digit in the number, and the number of balls in an urn represents how much greater than 1 that specific digit is. The number of possible positive integers is the number of ways to insert n balls into $8 - n$ urns for $n \in \{0, \dots, 7\}$. This equals $\binom{7}{7} + \cdots + \binom{7}{0} = 2^7 = 128$.

24) 120 is given in radians, not given in degrees, so there is no closed form for the expression.

25) $x_{n+2} = 2x_{n+1} + 3y_{n+1} = 2(2x_n + 3y_n) + 3(x_n + 2y_n) = 7x_n + 12y_n = 4x_{n+1} - x_n$. This corresponds to the characteristic equation $x^2 - 4x + 1 = 0$, which has roots $2 \pm \sqrt{3}$. Thus, $x_n = \alpha(2+\sqrt{3})^n + \beta(2-\sqrt{3})^n$. Setting $n = 0$ and $n = 1$, a system of equations $\alpha + \beta = 2$ and $\alpha(2+\sqrt{3}) + \beta(2-\sqrt{3}) = 7$ is obtained. The second equation simplifies to $\alpha - \beta = \sqrt{3}$, yielding the solution $\alpha = \frac{2+\sqrt{3}}{2}$ and $\beta = \frac{2-\sqrt{3}}{2}$. Thus, $x_n = \frac{(2+\sqrt{3})^{n+1}+(2-\sqrt{3})^{n+1}}{2}$. $A = 2, B = 1, C = 3$, and $D = 2$. $x_2 + y_1 + x_3 + y_2 = 26 + 4 + 97 + 15 = 142$.

26) Application of the periodicity of the last digit of the result from multiplying a number by itself, it can be found that the sequence of last digits of n^{n+3} are periodic with period 20, and that the last digit of the sum of each number in one period is 4. $Y = 2020$ would be 101 periods, which ends in a 4. $Y = 2018$ takes the last two terms from a completed period (1 and 0), so the sum ends in a 3.

27) Using the Method of Finite Differences, the following is obtained.

2015	2016	2017	2018	2019	2020
2	2	1	−1	−4	−8
0	−1	−2	−3	−4	
−1	−1	−1	−1		

28) The sum is equal to $b(a+1) + \frac{ca(a+1)}{2}$. This becomes $2b + ca = \frac{4036}{a+1}$ with manipulation. This creates five possibilities for a: 1, 3, 1008, 2017, and 4035. The following chart shows the number of possible solutions there are for each value of a.

a	Eq.	Sol.
1	$2b + c = 2018$	1008
3	$2b + 3c = 1009$	168
1008	$2b + 1008c = 4$	0
2017	$2b + 2017c = 2$	0
4035	$2b + 4035c = 1$	0

There are 1176 solutions. The sum of the digits of 1176 is 15.

29) Set $f(x) = \cos x$ in the difference quotient: $\lim_{n\to 0}\frac{\cos(x+n)-\cos x}{n} = \lim_{n\to 0}\frac{\cos x\cos n-\sin x\sin n-\cos x}{n} = \lim_{n\to 0}\frac{\cos x(\cos n-1)}{n} - \lim_{n\to 0}\frac{\sin x\sin n}{n} = \cos x\cdot\lim_{n\to 0}\frac{\cos^2 n-1}{n(\cos n+1)} - \sin x\cdot\lim_{n\to 0}\frac{\sin n}{n} = -\cos x\cdot\lim_{n\to 0}\frac{\sin^2 n}{n(\cos n+1)} - \sin x = -\cos x\cdot\lim_{n\to 0}\frac{\sin n}{n}\cdot\lim_{n\to 0}\frac{\sin n}{\cos n+1} - \sin x = -\sin x$.

30) The rate is the same for both performances, so it takes an equal time (60 minutes) for Rex Duodecim Angelus to be played.

1. E
2. B
3. B
4. A
5. D
6. C
7. D
8. B
9. A
10. C
11. A
12. D
13. D
14. A
15. B
16. D
17. A
18. D
19. A
20. C
21. C
22. C
23. A
24. E
25. C
26. A
27. C

28. B
29. C
30. B